ENDOGENOUS GROWTH, POPULATION GROWTH
AND THE REPUGNANT CONCLUSION

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Working Paper n. 2010-14
APRILE 2010
Endogenous Growth, Population Growth and the Repugnant Conclusion

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Abstract

This paper studies the impact of endogenous population change on economic growth, analyzing the simplest optimal endogenous growth model, an AK type model, driven by human capital accumulation. We show that in steady state both demographic change and economic growth are constant, but the rate of these growth can be positive, negative or null accordingly to parameter values. Population dynamics is determined by the difference between the stationary fertility rate and the exogenous mortality rate: if this is positive population size indefinitely increases, otherwise it reaches a stationary level, which can be positive (if the difference is null) or null (if it is negative). If fertility is strictly lower than mortality, population size will constantly decrease in finite time and we end up with a complete collapse of the economy, characterized by the total extinction of the population. We also analyze the problem of optimal population size and its relationship with growth. The seminal work of Parfit (1984) suggests that total utilitarianism leads to increase population size indefinitely, even if it the average welfare tends to zero. We show that in our model economy, under certain parametric conditions, the repugnant conclusion holds; in particular, this happens when consumption growth is negative and the stationary fertility rate is higher than the exogenous mortality rate.

Keywords: Economic Growth, Human Capital Accumulation, Endogenous Fertility, Optimal Population, Repugnant Conclusion

JEL Classification: O40, O41, J13

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†I wish to thank, without implicating, Raouf Boucekkine and Davide La Torre for useful advices and comments. I also thank the participants of the workshops held at the University of Auckland (Southern Workshop in Macroeconomics 2010, March 2010) and Victoria University of Wellington (15th Australasian Macroeconomics Workshop, April 2010) for helpful suggestions. As usual, errors and omissions are my own responsibility.
1 Introduction

The literature on population and economic growth is at least as old as economic science itself: Adam Smith and others before him understood that the relevant growth measure concerns per-capita rather than aggregate variables while Malthus dramatized the concept by identifying population as a possible threat for growth. However, despite the huge body of theoretical and empirical research, economists and demographers still do not have a shared view on the connections between population change and economic growth, as Bloom et al. (2003), clearly summarize: "...Though countries with rapidly growing populations tend to have more slowly growing economies..., this negative correlation typically disappears (or even reverses direction) once other factors... are taken into account".

Three main approaches have been proposed in the literature in order to study this issue: an optimistic, a pessimistic and a neutral view (see Bloom et al., 2003). The optimistic view (Kuznets, 1960 and 1967, and Boserup, 1989; most recent analysis can be found in Jones, 2001, and Tamura, 2002) considers population growth as a fuel for economic performance; this may be the result of knowledge production (since population is an input of this process, more researchers produce more knowledge\(^1\) or technical change (since population growth raises the returns to innovation). The neutralist view (Bloom et al., 2003) instead has empirical foundation and concludes there exists little cross-country evidence that population growth might either slow down or encourage economic growth, when other factors are taken into account. The most probably diffused one is pessimistic (Solow, 1956, Becker and Barro, 1988, and Barro and Becker, 1989) and sees population as detrimental for growth. This effect works through two different channels: in an economy with fixed resources and without any source of technological progress, the (food) production activity is overwhelmed by the pressures of population growth, and this can lead the available diet to fall below the subsistence level, lowering productivity growth rate (Malthus\(^2\), 1798); in an economy with rapid population growth, instead, a large part of investment is used to satisfy the needs of the growing population (investment-diversion effect - Kelley, 1988), rather than to increase per-capita capital endowments, leading to a negative impact on capital intensity. The proponents of this view base their argument on the idea that an increase in the population size leads to a dilution of reproducible resources.

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\(^1\)Jones (2005) summarizes the concept as: "... just as the total output of any good depends on the total number of workers producing the good, more researchers produce more new ideas. A larger population means more Mozarts and Newtons, and more Wright brothers, Sam Waltons, and William Shockleys"

\(^2\)Malthus (1798) famously concludes: "Taking the population of the world at any number, a thousand millions, for instance... the human species would increase in the ratio of 1, 2, 4, 8, 16, 32, 64, 128, 256, 516, etc. and subsistence as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, etc. In two centuries and a quarter the population would be to the means of subsistence as 512 to 10; in three centuries as 4096 to 13, and in two thousand years the difference would be incalculable". More recent studies supporting this conclusion can be found, for example, in Ehrlich (1968): "The battle ... is over. In the 1970s hundreds of millions of people are going to starve to death"
However, the literature concerning (optimal) growth and population change is quite limited and has mainly focused on the case of exogenous population change. Only few studies analyze this issue when demographic change is endogenous\(^3\), namely Barro and Becker (1989), Palivos and Yip (1993) and Razin and Sadka (1995). While the first paper considers a neoclassical framework, where parents care of their offspring’s future utility, permitting to aggregate a dynamic utility function, the others analyze endogenous growth models. But, while the aim in Palivos and Yip (1993) is to study how the choice of the Benthamite rather than the Millian criterion affects the outcome of the model, that of Razin and Sadka (1995) is to show that in steady state Parfit’s repugnant conclusion does not hold. Therefore, the analysis of the relationship between endogenous (optimal) growth and endogenous fertility is still an open question and deserves more attention.

The aim of this paper is filling this gap, studying the linkage between economic growth and population dynamics: in particular it analyzes the relationship between endogenous fertility choices and human capital accumulation. Since the debate about the impact of population change on the economy is still puzzling and a unique view has not been diffused yet, a normative issue automatically arises. Respect to previous works, our approach is therefore normative, aiming at characterize the possible outcomes arising in a framework of endogenous growth and endogenous population change. Moreover, we study how this is related to the optimal population size problem, which consists of determining the optimal number of lives in a population. We focus mainly on Parfit’s repugnant conclusion, analyzing under which conditions it can be consistent with an optimal growth model.

In particular, we consider an optimal growth model, driven by human capital accumulation, where both per-capita consumption and fertility rate are endogenous choice variables. The determination of the population growth negatively affects human capital accumulation because of the dilution effect (as in Bucci, 2008): an higher number of children lowers the quality level of the average individual in the economy and therefore it determines a cost for the society to bring the human capital level of newcomers up to the average level of existing population. It is quite standard to assume that this cost is linear in the fertility rate, even if a so simple specification risks to under estimate the complex interaction between many factors which ultimately determine the fertility rate. We consider instead that it is non-linear: this assumption has foundation in the empirical evidence on the relationship between population growth and economic growth. In fact, a diffused view sees this relationship as non-monotonic and the absence of monotonicity means that economic and population growth are non-linearly related.

The empirical literature is critical on the existence of a relationship between population growth and human capital accumulation. Many papers analyze it at households level, but empirical evidence supporting both the pessimistic and the optimistic view can be found (for a survey see Kelley, 1998).

\(^3\)Many papers study this topic with overlapping-generation models, but their aim is usually positive rather than normative. See for example the literature on the "quantity-quality trade-off"
Also at country level, empirical studies fail to show the existence of a clear effect of population on human capital accumulation (see Schulz, 1987; and Kelley, 1996), concluding that the sign of this relation is uncertain. For example, Kelley (1996) summarizing his findings states: "...the net impact of this quantity-quality tradeoff in educational achievements [...] is highly uncertain".

As previously mentioned, the same is true for the relationship between population growth and economic growth. In fact, several studies on cross-sectional data fail to find a significant correlation between the growth rate of population and per-capita output (see Bloom et al., 2003). In fact the neutralist view, widely diffused since the mid-1980s, arises as a result of such studies. The main conclusion of this theory does not concern the absence of a correlation between population change and economic performance but rather the presence of a different impact varying from country to country. In fact, while the National Research Council (1986) affirms on balance ... slower population growth would be beneficial to economic development of most developing countries, some World Bank economists suggest that in some countries bigger populations can boost economic growth (see Kelly and Schmidt, 1994, for a survey).

This idea is further analyzed and developed in Kelly and Schmidt (1995), which shows that the impact of population on the economy depends on the level of economic development: the impact of population growth is negative for less developed countries, while it is positive for developed ones. Therefore, this impact can change over time as the development proceeds. They clearly conclude: "...population growth is not all good or all bad for economic growth: it contains both elements, which can and [...] do changes over time". According to this result, therefore, the relationship between population and economic growth is non-monotonic. Non-monotonicity implies that such a relationship is non-linear: we formalize this by introducing a non-linear dilution function in the law of motion of human capital.

The paper proceeds in the following manner. In section 2 we analyze some of the most important topics concerning the problem of optimal population size, both from a philosophical and ethical point of view and from an economical one. The attention is focused on the choice of the social welfare function and its main effects on the models outcome, in particular on what Parfit (1984) defines as repugnant conclusion. In section 3 we explicitly introduce the model and we characterize the optimal paths, while in section 4 we perform steady state analysis. The model is characterized by a balanced growth path equilibrium (BGP), along which the fertility rate is constant; moreover, we show that in steady state Parfit’s repugnant conclusion holds under certain parametric conditions. Section 5 shows this results in a particular case of the model, that is when the dilution effect in human capital is quadratic. We study the main implications of such a formulation and show when, under general parametric values, the repugnant conclusion holds. Then, we perform a comparative statics exercise, underlying how population change (and therefore total welfare) can be affected through certain kind of policies. Section 6 instead concludes and suggests possible extensions for further research on the
topic.

2 The Repugnant Conclusion

The issue on optimal population size dates back to Wicksell. Later economists and philosophers were interested in the question raised firstly by Wicksell, that is, "what is the optimal, or the most advantageous, number of lives in a population under given circumstances?" The typical view on such a question is that the optimal population is the population that, given some predetermined conditions, ensures the largest social welfare. But this is not so easy as it may firstly seem; in fact, defining the social welfare function means to adopt a certain criterion, which implicitly requires to assign a value to human life.

On the philosophical side, the seminal work of Parfit (1984) has raised a warm debate on the problem of optimal population size. In fact, traditional ethical theories have counterintuitive and paradoxical implication with respect to fertility issues and moral duties towards next generations. For example, total utilitarianism, which aims to maximize the total well-being in the society, leads to Parfit’s Repugnant Conclusion: "for any perfectly equal population with very high positive welfare, there is a population with very low positive welfare which is better."^5

If you want to answer the question whether it is better to have 1 or 2 people in the population, you need before to understand what better means, and in particular to whom better has to refer to. Such an issue is what philosophers call person-affecting morality issues: an outcome can be better than another if it is better for at least one person. However this is not an easy evaluation: a larger population includes people who would not exist in a scenario with a lower population, and for these people the word better is not meaningful: the alternative for them is not to exist at all. Therefore, this kind of comparison implicitly requires to assign a value to the non-living state, raising not simple ethical problems.

As noticed before, determining the optimal population size generates the necessity of defining a

^4The problem of the optimum population is much older than Wicksell: even in Plato ("The number of our citizens shall be 5040 - this will be a convenient number... which can be divided by exactly fifty-nine divisors... will furnish numbers for war and peace, and for all contracts and dealings, including taxes and divisions of the land", Laws, 737-738) and Aristotle ("A great state is not the same thing as a state with a large population. But certainly experience also shows that it is difficult and perhaps impossible for a state with too large a population to have a good legal government", Politics, 1326) writings we can find discussions on this topic.

^5See Parfit (1984, p. 388). The formulation of the Repugnant Conclusion we present here is from Arrhenius (2000), which is more general than Parfit’s one, since Parfit does not require that people with very high welfare are equally well off. In fact, Parfit wrote: "For any possible population of at least ten billion people, all with a very high quality of life, there must be some much larger imaginable population whose existence, if other things are equal, would be better even though its members have lives that are barely worth living".
social welfare function. The standard approach deals with static framework\textsuperscript{6} where the number of agents is not a choice variable and does not vary through time. The ethical problem implied in the issue derives from the comparison of allocations with different population sizes, because it requires to evaluate the utility of people alive in one allocation but not in the other: this implicitly means evaluating the utility of not being born. Consider now the probably most known criteria\textsuperscript{7} that have been proposed: average utilitarianism and total utilitarianism.

In average utilitarianism, the objective is to maximize the average or per-capita welfare of the society (this is often associated with Mill, who used it as an argument for limiting the size of the optimal population). Implicit in this criterion is that the utility of not being born is equal to the average utility. In fact, considering two allocations, one in which there is one individual with a utility of two, and one in which a second person with utility of one is added to the previous one, average utilitarianism suggests that the first allocation has to be preferred, since average utility, coinciding with that of the single-living individual, is higher. But the addition of the second person in our society does not lead any member of the first allocation to be worse off: it only adds another person with utility of one. Preferring the first allocation means assuming that the second person is better off not being born: clearly, this is arbitrary.

In total utilitarianism, instead, the objective is to maximize total wellbeing in the economy, that is the average welfare multiplied by the size of population (this has been argued both by some ethicists, as Singer, and some economists, as Meade and Dasgupta, following the eighteen century philosopher Bentham). Implicit in this criterion is that the utility of not being born is zero (this is no more arbitrary than any other real number, as for example the average utility, as proposed by average utilitarianism). According to total utilitarianism, in the previous example the second allocation is to be preferred. Total utilitarianism is part of a more general class of welfare criteria known as critical-level utilitarianism (Blackorby, Bossert and Donaldson, 2005). With this criterion adding a person always improves welfare if his utility exceeds a critical level, which is zero in total utilitarianism; it therefore implies that population should be increased indefinitely, even if average utility may approach zero. This is what Parfit defined as repugnant conclusion, since it implies a very

\textsuperscript{6}It deals with what Dasgupta calls a Genesis Problem. "In the Genesis Problem there are no actual people. All persons are potential. In its purest form, the Genesis Problem asks how many lives there should be, enjoying what living standards" (Dasgupta 1993, p. 381). Dasgupta (1993, p. 385) argues that this is not the right approach to the problem, since the current generation is trying to choose the future number of people and their living standard. This is called the Actual Problem: the actual people are what count when determining the well-being of a household, not the potential people that might become part of the family later. Therefore, potential people have no special claim on the rest of the family, since they are not part of the family

\textsuperscript{7}Another very well-known criterion is the max-min principle (Rawls, 1971), which proposes to maximize the welfare of the poorest. However, it has not been used to any great extent in formal modeling of economic growth, so we do not analyze it
low standard of living. Average utilitarianism, instead, suggests that population should be as small as possible, since this maximizes average welfare: there is no space for the repugnant conclusion in average utilitarianism.

If the parents care (or the planner) of their offspring’s (next generations) future utility, then it is possible to aggregate a dynastic utility function. The dynastic utility function can be seen as a mix of the Benthamite and the Millian social welfare function, according to the weight attached to later generations utility. This allows us to extend the previous analysis also to a dynamic framework, where we need to assume discounted utilitarianism as optimality criterion, both in its average or total specification. Using discounted total utilitarianism, it is no longer certain that the size of population will be high and average utility will tend towards zero, both in an endogenous growth framework (Razin and Sadka, 1995, and Palivos and Yip, 1993) and in a neoclassical one (Dasgupta, 1969). Razin and Sadka (1995, pp. 175-179) show this in a setting with human capital, where human capital is produced sufficiently efficiently. Also Palivos and Yip (1993), demonstrating that the Benthamite criterion leads to smaller population size and higher economic growth, end up with the somewhat surprising result that the lower is the birth rate and the higher is consumption per capita. However, one could claim that it is not surprising that utility does not approach zero in these two examples since resources are unlimited. Nevertheless, Dasgupta (1969) finds, for a production function that is homogeneous of degree less than one and an explicit subsistence level, that in steady state population will be stationary and consumption per-capita will be above the subsistence level; this point to the fact that if a parent cares of his children, it is not optimal to give birth to so many children so that their utility approaches zero, and this is why the repugnant conclusion does not hold, even if resources are fixed.

Notice that Parfit’s conclusion is a critique to total utilitarianism, which is a standard assumption in macroeconomic theory. In an economic growth framework, it represents the case in which increasing population size is desirable while per-capita consumption is decreasing. In this paper, we study whether and how the repugnant conclusion can be compatible with an optimal endogenous growth model where fertility is endogenously determined.

3 The Model

The economy is closed and composed of households that receive wages and interest income, purchase the consumption good and choose how much consuming and how many children to have. Population coincides with the available number of workers, so that there is no unemployment and the labor supply is inelastic (no leisure-work choice), and it grows in accordance to household decisions. The

Barro and Becker (1989) show that finitely living consumers behave as infinitely living ones if each generation takes into account next one.
The aggregate production function uses human capital to produce one homogeneous final good, that can be consumed or invested in human capital. The accumulation of human capital depends on output, consumption and the quality level of the average individual in the society.

The representative household wants to maximize its lifetime utility function, which is the sum of its instantaneous utility function, which is assumed to be iso-elastic:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma},$$

where $\sigma \in (0, 1)$. It depends only on its individual consumption level (it is not interested in aggregate consumption, but only in per-capita consumption) and does not depend on the fertility rate (having children or not does not affect the utility level).

Population grows over time at a non-constant rate, given by the difference between the birth rate, $n_t$, and the constant and exogenous mortality rate, $d$:

$$\dot{N}_t = (n_t - d)N_t,$$

where both $n_t$ and $d$ are strictly positive.

The production function depends linearly on human capital:

$$Y_t = AH_t$$

and investments are used for the creation of new human capital, for replacing obsolete capital (depreciation rate is assumed to be null for simplicity) and for providing the same level of education to all newcomers:

$$I_t = \dot{H}_t + \delta H_t + \phi(n_t)H_t,$$

where we set $\delta = 0$. Therefore, the law of motion of human capital is given by the difference between production, the consumption level and quality level of the average individual in the population:

$$\dot{H}_t = Y_t - C_t - \phi(n_t)H_t,$$

where $\phi(n_t)$ represents the dilution function. The dilution effect in human capital accumulation represents the cost of bringing the level of human capital of the newcomers up to the average level of the existing population: population growth tends, ceteris paribus, to reduce the quality level of the average individual in the population (see Bucci, 2008).

It has often been assumed that a linear function can be used to describe this kind of effect. We
consider instead a non-linear function

\[ \phi(n_t) = a n_t^\eta \]  

where \( a > 0 \) and for simplicity, we assume the function to be convex, that is \( \eta > 1 \). This non-linear term relies on the empirical evidence on the relationship between economic growth and demographic change. As mentioned before, a diffused view considers this relation to crucially depend on the phase of economic development. This implies the dilution function, which aim is to capture such a relationship, should be non-monotonic: non-monotonicity clearly means it is non-linear.

The social planner maximizes total welfare in the society, that is, it maximizes the welfare of the representative agent multiplied by the population size, under the economy resource constraint, the law of motion of demography and the initial conditions for human capital and population:

\[
\max_{c_t, n_t} \int_0^\infty u(c_t) N_t e^{-\rho t} dt \\
\text{s.t.} \quad \dot{H}_t = A H_t - N_t c_t - a n_t^\eta H_t \\
\dot{N}_t = (n_t - d) N_t \\
H_0, N_0 \text{ given}
\]  

The planner objective function takes into account the size of current and future generations, showing inter-temporal altruism, represented by \( \rho \), the rate of time preference (the lower the rate of time preference, the higher the planner’s altruism towards later generations), and full intra-temporal altruism (it means that the weight assigned by the planner to each member of the same generation is the same: the weight of each individual is independent of the size of the generation).

### 3.1 Optimal Paths

From the social planner maximization problem, we can derive the Hamiltonian function:

\[
H_t(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} N_t e^{-\rho t} + \lambda_t [A H_t - N_t c_t - a n_t^\eta H_t] + \mu_t (n_t - d) N_t
\]

\[9\] Notice that in our specification, the dilution function is monotonic and this affects the economic growth rate, which results to be a monotonic function of population growth (fertility rate). Probably a better assumption would be a non-monotonic function, such that the dilution effect is decreasing for small \( n_t \) and becomes increasing for \( n_t \) higher than a certain values. However, such a different assumption would complicate too much the model, therefore it seems convenient to illustrate it in its simplest form. Moreover, the non-linearity assumption is relevant per se since it allows us to optimally endogenize fertility without introducing it as an argument of the instantaneous utility function.

\[10\] A large literature analyzing this issue with overlapping generation models exist. It is widely accepted the idea, supported by empirical evidence on the cross-sectional distribution of fertility and education, that population and human capital are related by the so-called “quantity-quality trade-off” (see Becker and Lewis, 1973; more recent works on the quantity-quality tradeoff includes Becker et al., 1990, and Tamura, 1994). Notice that in such a model, the stock of human capital represents the quality side of this relation, while the stock of population corresponds to the quantity side.
and the first order necessary conditions:
\[
\frac{\partial H_t}{\partial c_t} = 0 \rightarrow c_t^{-\sigma} N_t e^{-\rho t} = \lambda_t N_t \\
\frac{\partial H_t}{\partial n_t} = 0 \rightarrow \eta \lambda_t a_n t^{\eta-1} H_t = \mu_t N_t \\
\frac{\partial H_t}{\partial H_t} = -\dot{\lambda}_t \rightarrow \lambda_t [A - an_t^\eta] = -\dot{\lambda}_t \\
\frac{\partial H_t}{\partial N_t} = -\dot{\mu}_t \rightarrow \frac{c_t^{1-\sigma}}{1-\sigma} e^{-\rho t} - \lambda_t c_t + \mu_t (n_t - d) = -\dot{\mu}_t
\]

Together with the initial conditions \(H_0\) and \(N_0\), the state equations:
\[
\dot{H}_t = AH_t - N_t c_t - an_t^\eta H_t \\
\dot{N}_t = (n_t - d) N_t
\]

and the transversality conditions:
\[
\lim_{t \to \infty} H_t \lambda_t = 0 \\
\lim_{t \to \infty} N_t \mu_t = 0.
\]

Solving the system, we can obtain the optimal paths of consumption and fertility growth:
\[
\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma} [A - an_t^\eta - \rho] \\
\frac{\dot{n}_t}{n_t} = \frac{1}{\eta-1} \left[ \frac{c_t N_t}{H_t} \left[ 1 - \frac{\sigma}{\eta(1-\sigma)an_t^{\eta-1}} \right] \right]
\]

Equations (16) is constant unless the term representing the dilution effect in human capital accumulation and it shows an inverse relation between the growth rate of consumption and the fertility rate. In particular, a rise in the fertility rate leads to a non proportional reduction in the growth rate of per-capita consumption and this is due to the dilution effect in human capital accumulation, which is non-linear. Notice that, the growth rate of consumption is a concave function of the fertility rate and the fertility rate maximizing consumption growth results to be null.

Equation (17) instead depends on the ratio between aggregate consumption and human capital and on the rate of fertility. The signs of these relations cannot be determined a priori: in fact, they crucially depend on the term in the square brackets. It is interesting to notice that the choice of the consumption level, determining the size of the ratio term, affects the growth rate of fertility. This is not surprising since the lifetime utility function is formed in a setting of altruistic parents, that have to determine both the number of children and their welfare level.

The TVC (14) implies that the growth rate of human capital is bounded above:
\[
\gamma_H < A - an_t^\eta
\]

while the TVC (15) implies that the ratio aggregate consumption-aggregate human capital is positive.
4 Steady State Analysis

The growth rates of the per-capita consumption, human capital, population and fertility rate are:

\[
\gamma_c = \frac{1}{\sigma} [A - an_i^\eta - \rho] \quad (19)
\]

\[
\gamma_H = A - \frac{ctN_t}{H_t} - an_i^\eta \quad (20)
\]

\[
\gamma_N = n_t - d \quad (21)
\]

\[
\gamma_n = \frac{1}{\eta - 1} \frac{ctN_t}{H_t} \left[1 - \frac{\sigma}{\eta(1 - \sigma)an_i^\eta - 1}\right] \quad (22)
\]

We now analyze possible equilibrium paths, considering a balanced growth path, along which the growth rate of fertility is null. We distinguish between non-degenerate and degenerate balanced growth paths, since it simplifies the exposition of the results.

**Definition 1:** *(Balanced Growth Path, BGP)* a balanced growth path, BGP, or steady state equilibrium, \((\bar{c}, \bar{H}, \bar{N}, \bar{n}, \gamma_c, \gamma_H, \gamma_N, \gamma_n)\), is a sequence of time paths, \(\{c_t, H_t, N_t, n_t\}_{t \geq 0}\), along which all economic variables grow at constant rates. A BGP is said non-degenerate if \(c_t\) and \(H_t\) grow at non negative rates, while it is said degenerate (DBGP) if \(c_t\) and/or \(H_t\) grow at negative rates.

Along the BGP, the fertility rate, \(n_t\), must be constant, \(n_t = \bar{n}\): this means that the growth rate of fertility is null:

\[
\bar{n} = \left[\frac{\sigma}{\eta(1 - \sigma)a}\right]^{\frac{1}{\eta - 1}} \quad (23)
\]

and the growth rate of population is:

\[
\gamma_N = \bar{n} - d. \quad (24)
\]

Consequently, the growth rate of per-capita consumption is:

\[
\gamma_c = \frac{1}{\sigma} [A - a\bar{n}^\eta - \rho] \quad (25)
\]

and that of human capital is:

\[
\gamma_H = A - \frac{\bar{c}\bar{N}}{\bar{H}} - a\bar{n}^\eta \quad (26)
\]

since the growth rate of aggregate human capital must equalize the growth rate of aggregate consumption\(^{11}\) otherwise \(\gamma_H = -\infty\) or it would violate the TVC (14):

\[
\gamma = \gamma_H = \gamma_C = \gamma_c + \gamma_N. \quad (27)
\]

\(^{11}\)In fact, if \(\gamma_C > \gamma_H\) we would asymptotically have \(\gamma_H = -\infty\), while if \(\gamma_C < \gamma_H\), we would have \(\gamma_H = A - an_i^\eta\), violating equation (18)
This implies that per-capita variables, consumption and human capital, grow at the same rate $\gamma = \gamma_c$.

In order for endogenous growth to be verified, we need to have a lower bound for $A$ (only when the inequality is strict we have positive growth; when it holds as equality the long run growth is null):

$$A \geq a\pi^n + \rho,$$  \hspace{1cm} (28)

and in order to ensure bounded objective function, instead, we have an upper bound for $A$:

$$A < a\pi^n + \frac{\rho}{1 - \sigma} - \frac{\sigma(p - d)}{1 - \sigma}. $$  \hspace{1cm} (29)

Therefore, in steady state the fertility is constant, while the economic and population growth rate can be positive, negative or null:

$$\pi = \left[ \frac{\sigma}{\eta(1 - \sigma)a} \right]^{\frac{1}{(1-\sigma)}} $$  \hspace{1cm} (30)

$$\gamma = \frac{1}{\sigma} [A - a\pi^n - \rho] $$  \hspace{1cm} (31)

$$\gamma_N = \pi - d.$$ \hspace{1cm} (32)

The sign of the steady state economic growth rate depends on whether condition (29) is verified or not. If $A \geq a\pi^n + \rho$, per-capita consumption is non-decreasing, we have endogenous growth (which is strictly positive if the inequality is strict or null if it holds as equality) and the economy lies on the BGP. Instead, if $A < a\pi^n + \rho$, per-capita consumption decreases over time and the economy lies on the DBGP.

The steady state of population size, $N$, depends on the difference between the stationary fertility and the exogenous mortality rate. If the fertility is lower than mortality rate, the population growth rate is negative and in the long-run all individuals will disappear: the population will continue to decrease until its complete disappearance ($N = 0$) but this would lead to have an high rate of growth during the life of the economy. If birth and mortality rate perfectly offset, the population will reach a positive stationary equilibrium level, coinciding with its initial level ($N = N_0$). If, instead, fertility rate is higher than death rate, the population size will continue to rise ($N = \infty$), leading to a lower growth rate.

**Proposition 1**: along the (D)BGP, the following results hold:

(i) the stationary fertility level is a positive function of the inverse of the elasticity of substitution, $\sigma$, while it is a negative function of the dilution effect parameter, $a$;

12This result is due to the fact that the model does not show any transitional dynamics, as it will discussed at the end of this section
(ii) the growth rate of the economy depends negatively on the stationary fertility rate, $\bar{n}$;  
(iii) population growth is a positive function of the stationary fertility level, $\bar{n}$ and a decreasing function of the mortality rate, $d$.

Proof: The result just derives from the partial derivatives of (30), (31) and (32) respect to the main parameters. ■

Proposition 1 says that in our model economic growth is negatively affected by demographic growth (fertility), independently of the gap between fertility and mortality. Notice that such a result is a novelty in the literature: in standard growth models, the Benthamite criterion implies that the economic growth is independent of population variations; this result is due to the presence of non-linear dilution effects. Moreover, when demographic change is positive (null) population size continues to increase (reaches a stationary level) along the BGP, while when population growth is negative, its size converge to zero. Along the BGP, it can be optimal for the planner (if $\bar{n} < d$) to exploit an high growth and decreasing population in finite time and end up with a complete collapse of the economy, characterized by the full disappearance of the population.

Parfit (1984) defines as repugnant the outcome where an increase in population is accompanied by a decrease in per-capita consumption. Therefore along the BGP, where per-capita consumption is increasing, Parfit’s repugnant conclusion cannot hold. However, when condition (28) is not met, since per-capita consumption growth is negative (and the economy lies on the DBGP), the model is consistent with such an outcome if population growth is positive.

Proposition 2: along the BGP, Parfit’s repugnant conclusion does not hold; along the DBGP (where $A < a\bar{n}^\eta + \rho$), instead, if $\bar{n} > d$ then Parfit’s repugnant conclusion holds; otherwise, it does not.

Proof: the condition $\bar{n} > d$ ensures population size is indefinitely increasing in steady state while $A < a\bar{n}^\eta + \rho$ ensures consumption growth is negative (and therefore average utility is decreasing) in steady state. ■

Along the DBGP, when the repugnant conclusion does not hold, that is $\bar{n} \leq d$, total welfare tends to zero, since consumption (and therefore utility) asymptotically approaches zero and population size tends to a finite value (zero as well if $\bar{n} < d$ or a positive number if $\bar{n} = d$).

We can notice that in steady state, when the fertility rate is constant, the dynamic behavior of the economy is the same as in a standard AK model. Moreover, as in standard AK model, the economy lies along its (D)BGP since time 0. In fact, at time 0 if the fertility rate is chosen equal to $\bar{n}$, the growth rate of the economy, given is equation (31) is constant. Therefore, there do not exist any
transitional dynamics and the steady state outcome characterizes the entire dynamic path. Notice that also Palivos and Yip (1993) find that an AK-type model with endogenous fertility does not show any transitional dynamics. This means both population growth and economic growth are constant as in standard economic growth models, but the rate of these growth can be positive, negative or null accordingly to parameter values. If condition (28) is verified, we have endogenous growth and the economy lies on the BGP; otherwise consumption is decreasing and the economy lies on the DBGP. Equation (32) instead characterize the sign of population change, which depends on the difference between the stationary fertility rate and the exogenous mortality rate and ultimately determines the stationary population size.

5 A Specific Case: Quadratic Dilution Function

We now consider a specific case of the dilution function, which can be considered as a benchmark case, in order to exemplify the model and its steady state implications with refers to Parfit’s conclusion. In particular we consider the dilution function to be quadratic in the fertility rate, which corresponds to the specification, $\eta = 2$:

$$\phi(n_t) = an_t^2.$$ 

In such a case, along the BGP, the growth rate of the economy, population growth and the stationary fertility rate becomes:

$$\gamma = \frac{1}{\sigma} \left[ A - an^2 - \rho \right]$$

$$\gamma_N = \frac{n - d}{\sigma}$$

$$n = \frac{\sigma}{2(1 - \sigma)a}.$$ 

We now illustrate the growth rate of population implied by such a specification, under a given set of parameter values. In choosing such values we rely on existing empirical estimates or on baseline specifications coming from previous works. The main interest concerns verifying under which parameter values Parfit’s repugnant conclusion holds. Proposition 2 tells us the only case where this can arise is along the DBGP. Therefore, we concentrate only on such a case and for simplicity we set $A = 0$ (setting $A = 0.0001$ would not change the result: what matters is that the condition $A < an^2 + \rho$ holds, in order for the economy to lie on the DBGP). Since $\sigma \in (0, 1)$ for the fertility rate to be positive, we set it equal to 0.4 , following Arrow (1971), and Hansen and Singleton (1982,1984), who estimate the degree of relative risk aversion (RRA) to be lower than 1. We set $\rho = 0.04$, as

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13See Appendix A for more details
14Also other works confirm such estimates. For example, Epstein and Zin (1991) conclude the relative risk aversion can span the range from 0.4 to 1.4
in Mulligan and Sala-i-Martin (1993). We set \( d = 0.08 \), which represents the average among OECD countries of the United Nation (2009) crude death rate\(^{15}\). Then, we let \( a \) vary.

Figure 1: Population growth in steady state. The rate of population change is given by the difference between the stationary fertility rate (the blue curve) and the mortality rate (the red line). The figure shows the values of the stationary fertility rate, as the parameter \( a \) changes.

Under these assumptions boundedness of the objective function is always verified and the economic growth rate is negative. These considerations, jointly with Proposition 2, imply that Parfit’s repugnant conclusion holds if the rate of population growth in steady state is positive. This is given by the difference between the stationary fertility and mortality rates. The mortality rate is exogenous while the stationary fertility rate is endogenously determined in our model. In particular, \( \pi \) crucially depends on \( a \), the parameter representing the intensity of the dilution effect in human capital accumulation. Figure 1 shows how fertility changes as the parameter \( a \) varies; we let it vary between 0.5 and 5.5. If \( a \) is high the repugnant conclusion does not hold, while when it is low Parfit’s conclusion applies also to our framework. In fact, the population growth (the difference between the blue curve and the red line) is positive for any \( a \) lower than 4.17. This means that Parfit’s repugnant conclusion holds if the intensity of the dilution effect is not too high, otherwise it does not. In such a case, however, population size decreases over time and it leads asymptotically the population to vanish, and therefore the economy to collapse.

In our model economy, along the DBGP, population growth plays a central role in determining the total welfare in the society. Average welfare is constantly decreasing and therefore total welfare crucially depends on population growth. If it is positive, population size increases indefinitely and this can compensate the constant decline in average utility; if it is negative or null, total welfare

\(^{15}\)The crude death rate is the number deaths per thousand persons in the same year, based on mid-year population
converges to zero asymptotically while population size asymptotically converges to zero as well in
the former case or to a positive constant in the latter one. Therefore, policies oriented to affect
population change are crucial in order to reach any desired level of social welfare. This can be done
especially with two different kind of policies: one consists of manipulating the mortality rate while
the other deals with the dilution effect parameter. In the last part of this section, we performs a
comparative statics exercise, studying the impact of such kind of policies on the economic performance
and population growth.

5.1 Changes in the Mortality Rate
Suppose a shock decreases permanently the exogenous mortality rate, represented by the parameter
d, without affecting any other parameters of the model. Some economic shocks amount to changes
in the mortality rate; for example, some economic policies (supposing they can be achieved at zero
cost), like public expenditures in health or incentives to private health care, can modify the mortality
rate.

Suppose a new policy has just been introduced and its effect is to lower the mortality rate,
affecting therefore the population growth. If the economy were initially along the DBGP, this shock
has the effect of shifting the economy from a DBGP to another one. Along the new DBGP the
economic growth rate and the stationary fertility rate would remain unaffected while net population
growth changes.

If originally the population growth rate were positive, along the new DBGP population growth
would be higher and this leads to a faster increase in the population size. The same comments apply
if the original population growth rate were null.

If instead originally the population growth were negative, along the new DBGP population growth
would increase but it could be positive, negative or null, in accordance to the magnitude of the change.
If the new population growth were positive we would be driven back in the previous case. If it were
still negative, population would continue to decrease at an higher rate implying a faster decline in
the population size. If instead it were null, population size would reach a stationary level.

Therefore the introduction of a new policy aiming to increase public or private spending in health
care, through the effect of lowering the mortality rate, can be used in order to reach any desired
level of population, without encountering any change in the economic growth rate. Notice that the
repugnant conclusion can arise or disappear in the economy as a result of such a kind of shocks.

Suppose instead a shock, like the appearance of a new disease, increases the mortality rate. Its

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16Short-run and long-run effects coincide, as in the standard AK model: every shock in the economy translates in
a jump of the (economic and population) growth rates, of the stationary fertility level or both. In fact, at any shock
the fertility rate can be adjusted in order to lie directly on the new (D)BGP
effect is just the opposite of that of a policy promoting health care: along the new DBGP, the economic growth rate remains unaffected while the population growth changes have opposite signs respect what just discussed.

5.2 Shifts in the Dilution Parameter

Some economic policies (at zero cost) can affect the parameter $a$, representing the intensity of the dilution effect. Such a policy can be, for example, the introduction of public expenditure to improve the efficiency of teachers and teaching and its effect would be to decrease the cost for the society to bring children up to average level of human capital (decreasing $a$).

Suppose such a disturbance affects the economy. If the economy were initially along the DBGP, along the new one the economic growth, population growth and the steady state fertility level change. A drop in $a$ leads first of all to a shift from a fertility rate to an higher new one, while the fertility growth continues to be null. This implies an higher population growth and a lower economic growth.

Also the introduction of a policy aiming to affect the efficiency of teachers can be used to affect the population size and the existence of the repugnant conclusion. But differently from a policy oriented to the health care, it can also be used to foster growth.

6 Conclusion

Really few papers analyze the relationship between economic and population growth, when demographic change is endogenously determined. This paper try to fill this gap by analyzing an optimal growth model, driven by human capital accumulation, where fertility choice is endogenous and it is taken jointly with consumption decisions: not only consumption choices, but also fertility ones affect human capital accumulation through a dilution effect. Increasing the population size dilutes human capital, that is, the larger the population, the lower average human capital. This is due to the fact that newcomers human capital cannot be brought up to the average level with no costs.

We show that in steady state both population growth and economic growth are constant as in standard economic growth models, but the rate of these growth can be positive, negative or null accordingly to parameter values. If per-capita consumption is increasing, we have endogenous growth and the economy lies on the BGP; otherwise consumption is decreasing and the economy lies on the DBGP. Population dynamics is determined by the difference between the stationary fertility rate and the exogenous mortality rate: if this is positive population size indefinitely increases, otherwise it reaches a stationary level, which can be positive (if the difference is null) or null (if it is negative). In this last case, where fertility is strictly lower than mortality, population size will constantly decrease in finite time and we end up with a complete collapse of the economy, characterized by the total
extinction of the population. Moreover, the planner can intervene in the economy to avoid this result, through policies oriented to affect the fertility or the mortality rate. For example, the fertility rate can be increased through policies oriented to affect the efficiency of teachers; the mortality rate, instead, can be lowered increasing public expenditures in health or incentives to private health care.

We also study the implications of the model for the optimal population size problem, which deals with the identification of the most advantageous number of lives in a population and strictly relates to the choice of the social welfare function. In a static framework, total utilitarianism implies Parfit’s repugnant conclusion: it is always worthwhile to add a person in the society if his lifetime utility is higher than zero; this means that a society would indefinitely increase its population size, even if the average welfare is close to zero. This according to Parfit is repugnant since it leads to a mass of poverty. In a dynamic framework, we cannot use any more total utilitarianism but we need to rely on discounted total utilitarianism. According to this criterion, the repugnant conclusion does not hold, both in an neoclassical context, as demonstrated in Dasgupta (1969), and in an endogenous growth framework, as in Palivos and Yip (1993) and Razin and Sadka (1995). We show that in our model, where the dynamic social welfare function is defined accordingly to the discounted total utilitarianism criterion, in steady state along the DBGP (when consumption growth is negative), the repugnant conclusion holds if the stationary fertility rate is higher than the exogenous mortality rate.

For further research we suggest to extend the analysis along two different directions: considering a multi sectorial economy and the effect of different welfare criteria. In our model, population growth, endogenously determined by fertility choices, explicitly shows only a negative effect in the economic dynamics through its link with consumption choices. In fact, in our one sector economy, an increase in the growth of population lowers human capital accumulation, which is the engine of growth. Therefore, the result of our model, that is decreasing the number of persons in the economy is positive for growth, is implied in such a formulation. Probably a better specification of the model could be a two sectors economy, á-la Uzawa-Lucas, where population size negatively affects human capital accumulation, still representing the engine of growth, but has also positive effects on the accumulation of physical capital. Such a model could probably be more realistic in describing the role of row population: a higher number of persons leads to accumulate more physical capital, but it also has high costs in terms of the necessity to bring the under average skilled people up to the average level in order to promote growth. This double effect of population size on the economy can generate a more complicated dynamics, whose net effect can be not obvious and more interesting.

Moreover, in our model, the social welfare function is defined accordingly to the Benthamite criterion, as in standard economic growth theory. As previously seen, depending on the degree of altruism of the planner, the dynastic welfare function can be represented as a mix of the Benthamite and Millian criterion. It can be interesting to investigate whether the choice of the welfare function affects the outcome of the model, in particular how this relates to population and economic growth.
A  On the Transitional Dynamics

The four dimensional dynamic system in \((c_t, n_t, H_t, N_t)\), given in equations (19) - (22), can be reduced in a two dimensional one, by introducing the intensive variable \(\varphi_t = \frac{c_t N_t}{H_t}\), and studying the planar system in \((\varphi_t, n_t)\):

\[
\dot{\varphi}_t = \frac{1-\sigma}{\sigma} \left(A - an_t^\eta\right) - \frac{\rho}{\sigma} + (n_t - d) + \varphi_t \\
\dot{n}_t = \frac{1}{\eta - 1} \varphi_t \left[1 - \frac{\sigma}{\eta(1-\sigma)an_t^{\eta-1}}\right]. \tag{33}
\]

The non trivial steady state is \(E = (\varphi, \pi)\), where:

\[
\varphi = \frac{\rho}{\sigma} - \frac{1-\sigma}{\sigma} \left(A - a\pi^\eta\right) - (\pi - d) \tag{35}
\]

\[
\pi = \left[\frac{\sigma}{\eta(1-\sigma)a}\right]^{-\frac{1}{\eta-1}} \tag{36}
\]

Notice that \(\pi\) is positive if \(0 < \sigma < 1\) while \(\varphi\) is if \(\pi < \frac{\eta(\rho + \sigma d - (1-\sigma)A)}{\sigma(\eta-1)}\). This last condition is automatically ensured\(^{17}\) by equation (29). Notice that, since \(\pi > 0\), we need to have \(\frac{\rho + \sigma d - (1-\sigma)A}{\sigma(\eta-1)} > 0\), that is \(A < \frac{\rho + \sigma d}{1-\sigma}\).

We can study the stability of the steady state, by linearizing:

\[
J(\varphi_t, n_t) = \begin{bmatrix}
\frac{1-\sigma}{\sigma} \left(A - an_t^\eta\right) - \frac{\rho}{\sigma} + (n_t - d) + 2\varphi_t & \varphi_t \left[1 - \frac{\eta(1-\sigma)a}{\sigma(\eta-1)} n_t^{\eta-1}\right] \\
\frac{1}{\eta - 1} \left[n_t - \frac{\sigma}{\eta(1-\sigma)an_t^\eta}\right] & \frac{1}{\eta - 1} \varphi_t \left[1 - \frac{\sigma(2-\eta)}{\eta(1-\sigma)n_t^{\eta-1}}\right]
\end{bmatrix} \tag{37}
\]

and evaluating the Jacobian matrix at steady state \(E\):

\[
J(E) = \begin{bmatrix}
\varphi & 0 \\
0 & \varphi
\end{bmatrix}. \tag{38}
\]

Since \(\varphi\) is positive, it is straightforward to notice that both eigenvalues are positive, implying that the system never reaches the steady state (namely its BGP or DBGP), unless the initial choice for \(n_t\) is such that the fertility rate coincides with its stationary level from time 0. Therefore, the economy lies immediately on the (D)BGP and the economy does not show any transitional dynamics. Since \(n_0 = \pi\), the model behaves as a standard AK model and the steady state outcome of the model describes its entire dynamic path.

References


\(^{17}\)The condition to impose boundedness of the objective function, given by equation (29) is equivalent to that for ensuring positivity of the consumption-human capital ratio


