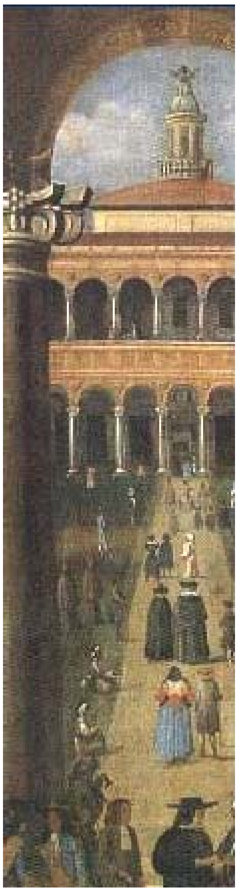

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On the definition and estimation of the value of a "statistical life"*

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Abstract

The presentation will discuss the economic meaning of the concept of the Value of a Statistical Life (VSL) and review some of its properties. In particular, the age pattern of the VSL is considered. The presentation will also review recent estimates of the magnitude of a VSL taken from different countries and cultures.

The value of preventing a fatality or (saving) a statistical life is an important question in health economics as well as environmental economics. This paper reviews several of the issues discussed in the literature. For example, how do we define the value of a (statistical) life? Are there really strong theoretical reasons for believing that the value of a life declines with age? The paper derives definitions of the value of a statistical life in both single-period models and life-cycle models. Models with and without actuarially fair annuities are examined, as well as the age-profile of the value of a statistical life.

Introduction

In many cases, such as environmental pollution and new medical treatments we are interested in estimating the benefits and costs of measures reducing the risk of death. A quite natural way of formulating the problem is in terms of the benefits and costs of a measure expected to save one life. If the value of saving one (statistical) life exceeds the costs incurred, undertaking the measure would seem worthwhile. It should also be mentioned that nowadays many authors seem to prefer to speak of the value of preventing a fatality rather than the value of a statistical life. In this paper, I will stick to the old fashioned terminology, however.

Even if we set aside all the problems faced in arriving at a reasonable empirical estimate of the value of preventing a fatality, many questions still remain. For example, how do we define the value of a (statistical) life? Are there really strong theoretical reasons for believing that the value of a life is declining (or increasing) with age? The purpose of this paper is to review definitions of the value of a statistical life in single-period as well as in life cycle models. I will not here take the risk of digging into who “invented” the concept of the value of a statistical life but paper 2 in Jones-Lee (1976) provides a fascinating review of contributions and views up to the early 1970s; in addition Jones-Lee (1974, 1976) seems to be the first researcher to derive a stringent definition of the value of a statistical life; Shepard and Zeckhauser (1982, 1984) and Arthur (1981) provide other early contributions[1].

There are good reasons for exploring several different models, for example with respect to the availability of actuarially fair annuities. First, we do not know what model people have in mind when making decisions. Therefore, the mechanical use of one definition or another of the value of a statistical life in a cost-benefit analysis of a measure preventing a fatality might cause a seriously biased estimate of benefits. Second, the institutional set-up varies between countries and a definition appropriate for one country might be less relevant for another.

The paper is structured as follows. Section 1 derives definitions of the value of a (statistical) life within two different single-period models. One definition refers to the case with no atemporal equivalent of an actuarially fair annuity (and no bequest motive). The second definition refers to the case where the wealth of a deceased is transferred to the survivors. Thus, there is a kind of (inverse) life insurance. Section 2 is devoted to a discussion of the definition of the value of a statistical life in a life-cycle model without actuarially fair annuities, while Section 3 considers the case where such annuities are available. An analysis of the age-dependency of the value of a statistical life is found in Section 4. A brief summary of the empirical evidence is contained in Section 5. A few concluding remarks can be found in Section 6.

1. The value of a statistical life: The single-period case

Throughout this paper, I will consider individuals that derive utility from consuming a single commodity if alive. The probability of survival is denoted μ, i.e. a fraction $1-\mu$ will die. Individuals are assumed to act as if they maximise their expected utility. In the single-period case, their expected utility is defined as follows:

$$U^E = \mu f(c) + (1-\mu)u^d \qquad (1)$$

where $f(c)$ is the utility enjoyed if alive, c denotes consumption if alive, and u^d denotes a fixed and finite level of "utility" assigned to the state dead; see, for example, Jones-Lee (1976) or Rosen (1988) for details. Deducting u^d from the utility derived in each state of the world in equation (1) yields $V^E=\mu u(c)$, where $u(c)=f(c)-u^d$.

Each individual is endowed with wealth k. Two different assumptions are employed with respect to ownership of k if the individual dies. According to the first variation wealth is passed on to the individual's heirs (who are not further considered here). Therefore, the budget constraint of a survivor is $k=c$, where the price of the single good is normalised to unity. Expected utility as a function of wealth is defined as $V^E=\mu u(k)$.

According to the second variation, borrowed from Rosen (1988), the wealth of a deceased person is transferred to those surviving. Since a fraction $1-\mu$ dies, each survivor will receive $k(1-\mu)/\mu$. Conditional on survival, the budget constraint is $k/\mu=c$. In this case, the expected utility as a function of wealth is defined as $V^E=\mu u(k/\mu)$.

Let us first examine the case with no atemporal equivalent of an actuarially fair annuity. Expected utility is defined as follows:

$$V^E = \mu u[k - CV(\mu)] \tag{2}$$

where $CV(\mu)$ is a payment, and $CV(\mu)=0$ initially.

Consider a small increase in the survival probability μ. Using equation (2), the WTP for such a risk reduction is defined as follows:

$$u(k)d\mu - \mu u_k(k)dCV = 0 \tag{3}$$

where $\mu u_k(.)=dV^E/dk$ is the expected marginal utility of wealth/income, and dCV is a payment such that the individual remains at the initial level of expected utility following a small increase $d\mu$ in the probability of survival.

Thus I have defined the WTP for a risk reduction. The value of a (statistical) life remains to be defined, however. Rosen (1988, p. 287) defines the *value of a life* as the marginal rate of substitution between wealth and risk, i.e.:

$$MRS_{k,\mu} = \frac{\partial V^E / \partial \mu}{\partial V^E / \partial k} = \frac{dCV}{d\mu} \tag{4}$$

Jones-Lee (1991, 1994) defines the value of *statistical* life as the population mean of $MRS_{k,\mu}$. Since I consider a cohort of ex ante identical individuals (facing identical risk reductions), I will here interpret equation (4) as providing a definition of the value of a statistical life[2] (VSL). Thus, we have the following definition of the VSL:

$$\frac{u(k)}{\mu u_k(k)} = \frac{1}{d\mu} dCV \tag{5}$$

where $\mu u_k(.)=dV^E/dk$ is the expected marginal utility of wealth. The left-hand side expression in equation (5) yields the gain in expected utility due to a small risk reduction converted from units of utility to monetary units by division by the expected marginal utility of wealth. The right-hand side expression in equation (5) yields the WTP for a risk reduction saving $d\mu$ lives multiplied by $1/d\mu$. Thus, the right-hand side expression yields the WTP for a measure expected to save one life.

Next, let us turn to the case where a survivor gets a tontine share[3]. Drawing on Rosen (1988), expected utility in this case is equal to:

$$V^E = \mu u\left[\frac{k - CV(\mu)}{\mu}\right] \tag{6}$$

where $CV(\mu)$ initially is equal to zero.

After straightforward calculations, the value of a statistical life is defined as follows:

$$\frac{u(.)}{u_k(.)} - c = \frac{1}{d\mu} dCV \tag{7}$$

where where $u_k(k/\mu)=dV^E/dk$ denotes the expected marginal utility of wealth, and $c=k/\mu$. In this case, the value of consumption is deducted from the monetary value of the direct gain in expected utility if a life is saved; the initial survivors will get fewer transfers from deceased individuals when the probability of death declines.

Equation (7) captures the value of unintended bequests. Therefore, this variation might seem more useful than equation (5) if the ultimate goal is to undertake a social cost-benefit analysis. In fact, the rule stated in equation (7) comes quite close to the rule generated by a simple single-period model with intentional bequests; see, for example, Johansson (2202c) for details. However, if people express altruism, for example toward other household members the outcome is changed. In a social cost-benefit analysis the WTP for altruistic motives would have to be added. Therefore, it is not entirely self-evident that the rule in equation (7) is more useful than the rule in (5) if the purpose is to undertake a cost-benefit analysis. For further discussion of the concept of altruism, see Jones-Lee (1991).

Next, I turn to life cycle models where actuarially fair life-assured annuities are available and not available, respectively. This seems to be a legitimate approach since empirical estimates of *dCV* might refer to either of the two models. In particular, if survey methods such as contingent valuation are used to collect information on the WTP for risk reductions, we do not necessarily know what model a respondent might have in mind.

2. A life-cycle model without life insurance

In this section, a life-cycle model where individuals face age-specific death rates replaces the single-period model. However, individuals are still assumed to derive utility from the consumption of a single commodity. Therefore, instantaneous utility at age t is equal to $u[c(t)]$. For simplicity, the utility discount rate $\theta \geq 0$ is assumed to be age-independent. The hazard rate $\delta(t)$, which yields the conditional probability of death in a short time interval $(t,t+dt)$, is assumed to be non-decreasing in age.

The remaining expected present value utility, given the survival of an individual until age τ, is defined as follows:

$$U_\tau^E = \int_\tau^\infty u[c(t)]e^{-\theta(t-\tau)} \frac{e^{-\int_0^t \delta(s)ds}}{e^{-\int_0^\tau \delta(s)ds}} dt = \int_\tau^\infty u[c(t)]e^{-\theta(t-\tau)} \mu(t;\tau)dt \quad (8)$$

where $\mu(t;\tau)$ denotes the probability of becoming at least t years old, conditional on surviving until the age of τ years. The consumption path is chosen so as to maximise (8), subject to the dynamic budget constraint stated in equations (A.1) in the Appendix. The individual has a capital income, i.e. interest on his wealth, and a wage/pension income. If less (more) than current income is spent on the single consumption good, then the individual will have a positive (negative) net accumulation of wealth.

The question is how to define the value of a statistical life within this framework. For the moment, let us simply *assume* that this value is defined as follows:

$$\frac{\int_\tau^\infty u[c^*(t)]e^{-\theta(t-\tau)} \mu(t;\tau)dt}{\lambda^*(\tau;\tau)} = \frac{V(\tau)}{\lambda^*(\tau;\tau)} \quad (9)$$

where an asterisk denotes a value along the optimal path, $\lambda^*(\tau;\tau)$ is a costate variable (dynamic Lagrange multiplier) yielding the marginal utility of consumption at age τ[4], and $V(\tau)$ denotes the value function, i.e. the value function yields the expected remaining present value utility of a utility-maximising individual aged τ years (and can be interpreted as the intertemporal counterpart to the single-period indirect utility function). Equation (9) measures the expected remaining present value utility converted to monetary units by division by the marginal utility of consumption at age τ.

Equation (9) yields a definition of the VSL corresponding to the one (in the case without actuarially fair life-assured annuities) suggested by, for example, Shepard and Zeckhauser[5] (1984). If a measure, say medical or environmental, "saves" one life, the gain in expected present value utility is given by the value function $V(\tau)$. Dividing through by the marginal utility of consumption at age τ will convert the expression from units of utility to monetary units. Rosen (1988) defines the VSL as the marginal rate of substitution between risk and wealth. Such a definition results in equation (9) if the attention is restricted to drops in the hazard rate lasting over very short periods of time. This result will be demonstrated below.

Let us now address the question of how to find a way of measuring the VSL as defined in equation (9). As a first step, let us consider an infinitesimally small change in the hazard rate lasting over a certain interval of time (beginning at age τ). This change will affect the survivor function also beyond this time interval, since the survival probability at any particular point in time depends on the integral (sum) of all previous hazard rates. The maximal once-and-for-all WTP at age τ, here denoted $dCV(\tau)$, in exchange for an increase in remaining expected present value utility is given by the following equation[6]:

$$dV(\tau) = \int_{\tau}^{\infty} u[c^*(t)] e^{-\theta(t-\tau)} d\mu(t;\tau) dt - \lambda^*(\tau;\tau) dCV(\tau) = 0 \tag{10}$$

where $d\mu(t;\tau)$ is the change in the probability of survival at age t conditional on being alive at age τ.

One would like to transform this equation so that it reflects the monetary value of the value function, i.e. $V(\tau)/\lambda^*(\tau;\tau)$. Johansson (2001) shows that equation (10) cannot be used to arrive at an unbiased measure of $V(\tau)/\lambda^*(\tau;\tau)$, unless consumption is *constant* across the entire life cycle. The problem is that instantaneous utility cannot be factored out from the integral in equation (10) if optimal consumption is age-dependent. This prevents any attempts to manipulate equation (10) so as to yield a variation of equation (9). The reader is referred to Johansson (2002c) for details.

There is an interesting case, however, where we can arrive at an unbiased estimate of VSL even if optimal consumption follows a *non-constant* pattern across the life cycle. This is the case where the drop in the hazard rate lasts over a very short time interval. (Blomqvist (2002) has considered this kind of a "blip" case, but similar cases have also been considered by, for example, Shepard and Zeckhauser (1982, 1984) and Rosen (1988).)

A possibility is to model such a change in the survivor function as suggested in Johansson (2002a) and Johannesson et al. (1997). In this case, illustrated in Figure 1, there is a drop equal to $d\kappa$ in the hazard rate lasting over the interval [τ, $\tau+\varepsilon$] for an individual who has survived until age τ. At age $\tau+\varepsilon$, the hazard rate returns to its initial path[7].

I claim that this approach, developed in Johansson (2002c), is exact (in contrast to, for example, the approximations used by Shepard and Zeckhauser (1982, 1984), and Blomqvist (2002)).

Using this way of modelling a change in the hazard rate, equation (10) above would read:

$$d\kappa\int_{\tau}^{\tau+\varepsilon} u[c^*(t)]e^{-\theta(t-\tau)}(t-\tau)\mu(t;\tau)dt + \varepsilon d\kappa\int_{\tau+\varepsilon}^{\infty} u[c^*(t)]e^{-\theta(t-\tau)}\mu(t;\tau)dt - \lambda^*(\tau;\tau)dCV(\tau) = 0 \qquad (11)$$

Next, multiply through this equation by $1/\varepsilon d\kappa$. As $\varepsilon\to 0$, the first term in this reformulated version of equation (11) tends to zero. This result follows from L'Hôpital's rule; see Johansson (2002c). The second term in (11) multiplied by $1/\varepsilon d\kappa$ yields the expected remaining present value utility of a τ-year old person (since $\tau+\varepsilon\to\tau$ as $\varepsilon\to 0$), i.e. the value function $V(\tau)$.

Thus, in the case of a true "blip", i.e. where $\varepsilon\to 0$, equation (11) reduces to:

$$\frac{V(\tau)}{\lambda^*(\tau;\tau)} = \frac{dCV(\tau)}{\varepsilon d\kappa} \qquad (12)$$

This expression yields a seemingly conventional single-period definition of the VSL, equal to the WTP for a risk reduction multiplied by one over the risk reduction (adjusted for the duration ε of the drop in the hazard rate). This result also holds if consumption is age-dependent. Moreover, it can be shown that equation (12) reflects the MRS between initial wealth and initial risk. This result confirms Rosen's (1988) interpretation of the value of a (statistical) life.

If the individual is asked to pay for a change $d\kappa$ in the hazard rate lasting over a longer period of time than "ε goes to zero", the approach suggested above will obviously provide a biased estimate of the value of a statistical life. This is due to the fact that the assumption of the first term in the left-hand side expression of equation (11) being equal to zero cannot be defended if the drop $d\kappa$ lasts over a "longer" ("$\varepsilon>0$") period of time. However, the approach provides an *upper bound* for the "true" value of a statistical life, since the following result can be shown to hold:

$$\frac{V(\tau+\varepsilon)}{\lambda^*(\tau+\varepsilon;\tau+\varepsilon)} \leq \frac{dCV(\tau)}{e^{-r\varepsilon}\varepsilon d\kappa} \tag{13}$$

where, for simplicity, r is the constant market rate of interest. Thus, $e^{r\varepsilon}dCV(\tau)/\varepsilon d\kappa$ provides an upper bound for the monetary value of the expected remaining present value utility of a person surviving until the age of $\tau+\varepsilon$, i.e. $V(\tau+\varepsilon)/\lambda^*(\tau+\varepsilon;\tau+\varepsilon)$. This result holds regardless of if consumption is constant or non-constant over the life cycle.

In sum, using empirical data to arrive at an unbiased estimate of the VSL seems possible if the WTP-measure refers to a true blip in the hazard rate. However, if the drop in the hazard rate lasts for a longer period of time, say a year, the measure might be biased, unless optimal consumption is constant across the life cycle. Unfortunately, there seems to be no obvious way of stating whether the bias is "small" or "large".

3. A life-cycle model with actuarially fair life-assured annuities

This section assumes that there are insurance companies offering actuarially fair insurance, i.e. the intertemporal equivalent of a tontine. There is a large number of identical individuals of age t. Following Yaari (1965) and Blanchard (1985), it is assumed that these individuals will contract to have all of their wealth return to the insurer, contingent on their death. While alive, they will in exchange receive a return per (short) time period equal to the hazard rate times their wealth. In order to provide a simple illustration, let us assume that the hazard rate is age-independent, i.e. $\delta(t)=\delta$ for all t. Then the remaining life expectancy is equal to $1/\delta$ (independently of current age). If optimal wealth is also age-independent and equal to k, then the individual will receive an amount δk at each point in time (or rather over a short interval of time t, $t+dt$). Since his remaining life expectancy is equal to $1/\delta$, in total the individual can expect to receive $(1/\delta)\delta k=k$ from his insurance company. Therefore, if the contract specifies that the insurance company will receive his wealth, i.e. k, when he dies, then the contract is actuarially fair.

The utility maximisation problem for the case with actuarially fair life-assured annuities corresponds to maximising equation (8) subject to equations (A.3) in the Appendix.

In a model with this kind of "inverted" life insurance (and where the hazard rate is non-declining in age), it can be shown that optimal consumption increases (decreases) if the market rate of interest r exceeds (falls short of) the utility discount rate θ. Thus, consumption is independent of risk; risk is insured away. This is in sharp contrast to the case without insurance, where the rate of change of consumption is driven by the sign of $r-\theta-\delta(t)$; see Johansson (2002c) for details.

Let us now consider a change in the hazard rate modelled as an infinitesimally small drop $d\kappa$ during an interval ε (beginning at the current age τ). Since the individual receives a capital income related to his hazard rate, a change in the hazard rate will have a direct impact on his budget constraint. This is the main difference between this case and the one considered in the previous section; it can be seen by comparing the Hamiltonians in equations (A.2) and (A.5), respectively. Proceeding in the same way as in equation (11), and restricting attention to a blip, one obtains (as is shown in Johansson (2002c)):

$$\frac{V^a(\tau)}{\eta^{**}(\tau;\tau)} + \int_\tau^\infty \left[w(t) - c^{**}(t)\right] e^{-\theta(t-\tau)} \mu(t;\tau) dt = \frac{dCV^a(\tau)}{\varepsilon d\kappa} \tag{14}$$

where $V^a(\tau)$ denotes the value function, i.e. the expected remaining present value utility conditional on survival until age τ. The VSL-result stated in equation (14) parallels the result in Rosen's (1988) equation (32). It parallels equation (34) in Shepard and Zeckhauser (1984) if $r=\theta$; they only consider the case where optimal consumption is constant for all t.

The VSL-measure in (14) differs from the one stated in (12), since the expected present value of the difference between current income and current consumption enters (14) but not (12). The term in question reflects the fact that reducing the risk of death will also mean fewer transfers from deceased to survivors, at least if assets $k(\tau)$ at age τ are strictly positive. Whether assuming such a ("inverted") life insurance is realistic is another question. It might also be noted that the VSL as defined by (14) might be larger or smaller than the VSL obtained through (12); see Shepard and Zeckhauser (1984, p. 430) for details. This result is hardly surprising. After all, the two approaches draw on different assumptions with respect to the institutional set-up of the economy. Hence, one would expect them to generate different optimal paths for consumption, different utility levels, and different valuations of marginal additions to wealth.

Oftentimes, a survey method, such as the contingent valuation method, is used to collect information on the WTP for a risk reduction. Then, there is the question whether an individual calculates his WTP-measure according to the model in this section or the one introduced in the previous section (or some other model). Obviously, there is room for a bias in the cost-benefit analysis if the investigator is mistaken.

Finally, the reader should recall the assumptions used in obtaining the VSL-measure in (14). The drop in the hazard rate must last over a very short period of time for the approach to be unbiased. It is not self-evident that this assumption is reasonable in empirical applications, where risk reductions might last over considerable time intervals.

4. On the age-dependency of the value of a statistical life

An important issue for decision-makers is whether the value of a statistical life increases or decreases in age. Assuming that the value declines with age might seem quite reasonable. For example, the DG Environment of the EU claims that there "are strong theoretical and empirical grounds for believing that the value for preventing a fatality declines with age" (p.2). This seems to be a very common view both in Europe and in the US; see Evans and Smith (2006) for details. In this section the age-dependency issue is addressed.

Let us first consider the case without life insurance. According to equation (12), the monetary value of the expected remaining present value utility of a person having survived until age τ is defined as:

$$VSL(\tau) = \frac{V(\tau)}{\lambda^*(\tau;\tau)} \tag{15}$$

Differentiating this expression with respect to τ indicates whether the *VSL* is increasing or decreasing in age. One obtains:

$$\begin{aligned} \frac{dVSL(\tau)}{d\tau} &= \frac{V_\tau(\tau)}{\lambda^*(\tau;\tau)} - \frac{V(\tau)}{\lambda^*(\tau;\tau)}\frac{\lambda_\tau^*(\tau;\tau)}{\lambda^*(\tau;\tau)} = \\ &[\theta+\delta(\tau)]\frac{V(\tau)}{\lambda^*(\tau;\tau)} - \frac{u[c^*(\tau)]}{\lambda^*(\tau;\tau)} - \frac{V(\tau)}{\lambda^*(\tau;\tau)}\frac{\lambda_\tau^*(\tau;\tau)}{\lambda^*(\tau;\tau)} \end{aligned} \tag{16}$$

where a subscript τ refers to a partial derivative with respect to current age τ, and $\lambda_{\tau}^{*}(\tau;\tau)$ = $-[r-\theta-\delta(\tau)]\lambda^{*}(\tau;\tau)$; see Johansson (2002c). Whether the value of a statistical life is increasing or decreasing in age depends on several factors. The first term on the right-hand side of equation (16) yields the gain in expected present value utility (converted to monetary units) as the future comes closer when the age of the individual increases marginally. This is a pure "discounting" effect, since both θ and $\delta(\tau)$ actually work like discount factors. The second term on the right-hand side of equation (16) captures the loss in instantaneous utility (converted to monetary units) when the individual becomes marginally older. The third term captures the fact that the marginal utility of income is, in general, age-dependent. Recall that the age-pattern of $\lambda^{*}(\tau;\tau)$ is determined by the difference between the sum of the utility discount rate plus the hazard rate and the market rate of interest. In turn, an age-dependency of $\lambda^{*}(\tau;\tau)$ will affect the monetary value of the remaining expected present value utility when age is marginally increased.

In order to shed further light on the sign of equation (16), assume that the probability of death is age-independent, so that δ is a constant. Moreover, assume that $r=\delta+\theta$. Then, optimal consumption remains constant across the entire life cycle, i.e. $c^{*}(t)=c^{*}$ for all t. In this case, instantaneous utility is constant for all ages, i.e. $u[c^{*}(t)]=u[c^{*}]$ for all t. Therefore, $V(\tau)=u[c^{*}]/[\delta+\theta]$ and the first two terms on the right-hand side of (16) net out. Moreover, $\lambda^{*}(\tau;\tau)$ is age-independent when $r=\delta+\theta$. Thus, *VSL* is independent of age in the case considered where optimal consumption is constant across the entire life cycle. Next, let us consider the case where optimal consumption decreases with age, i.e. $r<\delta(t)+\theta$. Then, $[\delta(\tau)+\theta]V(\tau)<u[c^{*}(\tau)]$ and $\lambda_{\tau}^{*}(\tau;\tau)>0$. Therefore, in the case of decreasing optimal consumption across the entire life cycle, *VSL* declines with age. See Johansson 82002c) for details.

More generally, equation (16) indicates that *VSL* might be increasing, constant, or decreasing with age, depending on the age-pattern of optimal consumption. The value might also be increasing (decreasing) with age over a certain age interval and then decreasing (increasing). Therefore, claims that there are strong theoretical reasons for assuming that the *VSL* declines with age seem somewhat premature.

However, it remains to check whether the introduction of actuarially fair life-assured annuities will affect the age-dependency of the *VSL*. In this latter case, the *VSL* is defined as follows:

$$VSL^a(\tau) = \frac{V^a(\tau)}{\eta^{**}(\tau;\tau)} + \int_\tau^\infty \left[w(t) - c^{**}(t)\right] e^{-\theta(t-\tau)} \mu(t;\tau)dt \tag{17}$$

In this case, the following expression for the age-dependency of VSL^a is obtained:

$$\frac{dVSL^a(\tau)}{d\tau} = \left[\delta(\tau) + \theta\right] VSL^a(\tau) - \frac{u\left[c^{**}(\tau)\right]}{\eta^{**}(\tau;\tau)} - \left[w(\tau) - c^{**}(\tau)\right] - \frac{V^a(\tau)}{\eta^{**}(\tau;\tau)} \frac{\eta_\tau^{**}(\tau;\tau)}{\eta^{**}(\tau;\tau)} \tag{18}$$

where $\eta_\tau^{**}(\tau;\tau) = (\theta - r)\eta^{**}(\tau;\tau)$ as is shown in Johansson (2002c). The sign of $dVSL^a(\tau)/d\tau$ seems to be ambiguous. In other words, equation (18) provides no obvious age-pattern for the value of a statistical life. Once again, one must reasonably conclude that the value of a statistical life might be increasing, constant, or decreasing (or show a more complicated pattern) in age.

It can be shown that, VSL^a is age-independent if $r=\theta$, the hazard rate is age-independent, and non-capital income is constant across the life cycle; see Johansson (2002c). On the other hand, VSL^a declines with age, for example if the hazard rate increases with age (while $r=\theta$, and $w(t)=w,\ \forall t$). Thus, one can find cases where the value of a statistical life declines with age. However, according to the examination undertaken in this section, it seems far from self-evident that there are strong theoretical reasons for believing that *VSL* and/or VSL^a declines with age. Further theoretical (as well as empirical) investigations seem warranted. For example, it might be fruitful to look at cases where some age groups, not the least the very old, face borrowing constraints; see, for example, Leung (1994) for an analysis of the implications of borrowing constraints.

5. On the empirical evidence

There are quite a few studies aimed at estimating the value of a statistical life. There are two broad groups of approaches used in the associated attempts to estimate the WTP for a risk reduction. Indirect methods use actual behaviour in a market to (indirectly) infer the value placed on, say, a risk reduction. For example, given current market prices some people invest in a fire alarm in order to reduce the risk of a fatal fire (and prices vary over time and/or space). Others might be prepared to take a risky job in exchange for a higher wage. Such information can be used to estimate the WTP for a risk change. In contrast, direct methods draw on hypothetical survey questions. For example, a random sample of respondents might be asked of their maximal WTP for a new medical treatment. This example illustrates the contingent valuation method. Recently, a slightly different technique has become popular in both environmental and health economics. According to this technique the attributes of a commodity are varied. In order to illustrate, let us assume that the commodity is a particular make of a car with a certain number of airbags and horsepower. Next, the number of airbags is increased while the number of horsepower is decreased. The respondent is asked whether or not this change in attributes represents an improvement. This technique, known as conjoint analysis, can also be used to estimate a WTP-measure.

de Blaeij et al (2000) undertake a meta–analysis based on 25 previous empirical studies of the value of statistical life in road safety; in total the 25 studies report 71 estimates of the value of a statistical life that are used in the meta-analysis. Studies using direct methods as well as studies using indirect methods are represented in the meta-analysis[8]. Meta-analysis is devoted to the statistical analysis of previous studies. It is commonly applied in the health and medical sciences, where researchers pool raw data from different studies to examine the relationship between the health outcome variable and variables assumed to affect the outcome; see, for example, Mann (1994) for a discussion. The way meta-analysis is used here differs slightly from the traditional approach since the variable of interest is estimates of a variable, i.e. the value of statistical life, rather than raw data.

According to de Blaeij et al (2000, p. 4) "Meta-analysis is a methodology, comprising a vast array of statistical techniques, developed in order to systematically analyze differences between outcomes of studies, ultimately leading to a synthesis of the results". A meta-regression is typically based on some least square estimator of a relation $y=f(\Pi,X)+\varepsilon$, where y is a specific effect measure observed in the studies, Π is the specific underlying cause, X is a vector of moderator variables such as differences in research designs, time periods examined, and locations/countries covered, and ε is an error term. (de Blaeij et al (2000, p. 12).)

In the survey by de Blaeij et al (2000), the magnitude of the VSL varies from about 400,000 to 30 million in 1998 US dollars; in what follows all values are converted to 1998 US dollars. However, the distribution is skewed to the right. Most studies report values of less than $5 million. This is true for more than 50 out of the 71 estimates that are reported by de Blaeij et al (2000).

According to their meta-analysis there is a positive relationship between the initial risk level and the value of statistical life. They also find that studies using indirect methods arrive at lower estimates than studies using direct methods. This result confirms a quite common view among economists. They expect that methods based on hypothetical choices will typically fail to establish a binding budget constraint. A "soft" budget constraint will probably imply that the reported WTP overestimates the "true" WTP.

The meta-analysis of de Blaeij et al (2000) include both studies using direct and indirect methods. In contrast, the meta-analysis of Mrozek and Taylor (2002) is based exclusively on a particular kind of studies, namely (over 40) labour market studies[9]. Such studies are based on the hypothesis that employees are willing to give up income in exchange for improved workplace safety while employers are willing to pay higher wages to employees willing to accept high-risk jobs. The income-risk trade-off is typically examined by estimating some variation of the following equation:

$$w_i = \alpha_0 + \alpha_1 \pi_i + \alpha_2 In_i + \alpha_3 Comp_i + \alpha_4 SE + \varepsilon_i \qquad (19)$$

where w_i is the wage rate of worker i, α_j denotes the coefficient associated with variable j, π_i (In_i) is the probability of a fatal (non-fatal) accident of worker i, $Comp_i$ is the monetary compensation received in the event of a non-fatal accident, SE refers to (a vector of) socio-economic factors, and ε_i is an error term. Equation (19) assumes that individual worker data are available.

In equation (19), the coefficient α_1 captures the impact of an increased risk of a fatality on the equilibrium wage rate. Thus this coefficient can be used to recover the value of a statistical life[10]. However, one would expect that the risks of fatal and non-fatal work place accidents, i.e. π_i and In_i, are highly correlated. This kind of collinearity complicates the interpretation of α_1 and makes the estimation of the value of a statistical life shaky.

Mrozek and Taylor (2002) claim that it is reasonable to believe that the value of a statistical life is approximately $2 million (1998 US) dollars when evaluated at average workplace death risks of 0.0005. This value is below the one reported by Viscusi (1992, pp. 73-74, 1993). He claims that most of the reasonable estimates based on labour market studies are clustered in the $4 to $9 million range[11]. Mrozek and Taylor (2002) claim that their analysis suggests that value of statistical life estimates over $2 to $3 million are likely to reflect the lack of attention the previous literature has given to the control of unobserved determinants of wages at the industry level.

There are also quite a few studies using survey techniques in order to estimate the WTP for a risk reduction. Early such studies typically used open-ended WTP-questions, i.e. asked respondents of their maximal WTP for a particular risk reduction. In recent years closed-ended formats have become quite popular. According to this latter approach, respondents accept or reject to pay a specified amount of money in exchange for a risk reduction, but the amount varies between different (subsamples of) respondents. The contingent valuation method has been criticised because the choices are hypothetical ("hypothetical responses to hypothetical questions") rather than real. Moreover, respondents seem to have difficulties in understanding the meaning of small risks and risk reductions. Possibly due to these difficulties many studies report a lack of a scope effect, i.e. increasing the risk reduction does not increase the WTP.

Viscusi (1992, chapter 4) claims that most of the reliable studies independently of method used produce value of statistical life estimates that fall in the interval $4 to $9 million (in 1998 US) dollars. The meta-analyses by Desvousges et al (1995) and Miller (2000) suggest that $4 million is an appropriate value for the US, while Mrozek and Taylor (2002) arrives at a "best" estimate of about $2 million for the US[12]. The reader that is interested in comparisons across countries is referred to Miller (2000). Miller's Table 2 (on p. 177) show estimated values of a statistical life for 13 countries, ranging from about $0.65 million for South Korea to about $8.5 million for Japan.

The US Environmental Protection Agency (EPA) uses $6 million as the value of a statistical life in its analyses of regulatory issues (in late 1990's). This value is based on labour market studies and (a few) contingent valuation studies. (EPA assumes that the values produced by these studies are drawn from a Weibull distribution. The mean of the distribution of the value of a statistical life turns out to be $6 million.)

A recent study by the EU's DG Environment recommends the use of a value in the interval €0.9 to €3.5 million. The best estimate according to this study is a figure of around €1.4 million. Alberini et al. (2004) in their contingent valuation study of Italy, France, and the UK arrive at an interval of €1.1 to €2.3 million.

Turning to the age-pattern the theoretical results are mixed; from a purely theoretical point of view, the age-pattern of the value of a statistical life is ambiguous. Turning to the empirical evidence, there seems to be some support for the idea that the age-pattern of the value of statistical life shows a kind of “inverted-u” shape. The value increases with age to peak at some age (τ*) and then starts to decline as is illustrated in Figure 2.

This kind inverted-U shaped relationship was probably first obtained by Shepard and Zeckhauser (1982, 1984). They derived the relationship within numerical life cycle models with and without actuarially fair annuities[13]; see Figure 3 and Table 2 in Shepard and Zeckhauser (1984, p. 434). Jones-Lee et al (1985) report an inverted-U shaped age-pattern in a large-scale national contingent valuation study of the value of transport safety in the UK. However, some of their valuation questions imply a value of life that is either constant across ages or shows a weak tendency to fall with age. Johannesson et al (1997) in their Swedish contingent valuation study of the WTP for a new medical treatment also report an inverted-u shape for the age-pattern of the value of a statistical life. Kniesner et al. (2006) estimate an inverted-U shaped age pattern drawing on labour market data for the US.

It might be noted that in studies reporting an u-shaped age-pattern the value often seems to peak around the age of 40 years. Possibly, such an age-pattern reflects an earnings-pattern over the life cycle where income peaks around the age of 40. The pattern in question might also be related to the fact that most people around the age of 40 have young children. Hence they might feel that it is more important to reduce the risk of a fatality at this stage in life than at other stages in life. However, Smith et al (2001) come to the conclusion that the value of statistical life does not vary systematically with age up to age 65. They use the theory of compensating wage differentials to estimate the value of a statistical life of elderly workers in the US. Evans and Smith (2006) and Kniesner and Viscusi (2005) come of with the finding that VSL might increase in age. These seemingly conflicting results are hardly surprising. Recall that we in Section 4 did argue that the value of statistical life might increase or decrease with age, or be age-independent, or show some more complex age-pattern.

6. Concluding remarks

This paper has been devoted to an examination of different definitions of the value of a statistical life. Both single-period and intertemporal models have been examined in some detail. It is possible to generalise the simple single-period definition of the VSL to a life-cycle perspective. That is, even in an intertemporal model of utility maximisation one can in a sense speak of a programme saving, say, 1 life out of 10,000 and use the WTP for such a risk reduction to define the VSL.

However, the practical problem seems to be that these theoretical definitions must be based on "blips", i.e. drops in the hazard rate lasting over very short time intervals. Therefore, and according to this paper, one would expect real world computations of the VSL to be biased, in general, at least if such applications are based on the contingent valuation method. Typically, people are asked to value reductions in the hazard rate lasting over "long" (say a year?) periods of time. Unless optimal consumption remains constant over the entire life cycle, the resulting measures will not reflect the monetary value of expected remaining present value utility.

More generally, many changes in death risks are more or less permanent, for example due to long-run changes in the degree of pollution of air or water. Similarly, a new medical drug might have a long run impact on a patient's hazard rate. If we use the WTP for such long-term drops in the hazard rate in order to calculate the VSL, one would expect the resulting VSL-estimates to be seriously biased. Therefore, it is important to further examine if and how "parametric" changes in death risks can be used to estimate the VSL.

Moreover, the paper has explored the implications for the definition of the VSL of different assumptions with respect to the availability of actuarially fair life-assured annuities. In particular, whether the expected present value of remaining income less the expected present value of remaining consumption should be added to the value of saving a life depends critically on the availability of actuarially fair life-assured annuities. In empirical studies, it is of importance to check whether an estimate of the VSL assumes the presence of actuarially fair life-assured annuities. Otherwise, there is a risk of double counting, if the estimates are used for a cost-benefit analysis.

The model with actuarially fair life-assured annuities considered in Section 3 seems to generate a VSL-definition that comes quite close to the cost-benefit rule derived by Arthur (1981). He used an intergenerational general equilibrium model with population growth to derive the value of a statistical life. Therefore, it seems as if the approximation provided in (14) is more useful than (12) for cost-benefit analysis. However, it should be stressed that the presence of bequest motives and (one-sided or double-sided) altruism might change this conclusion. Therefore, deriving cost-benefit rules for dynamic economies with population growth seems to be an important question[14].

The age-dependency of the VSL has also been analysed in this paper. It is sometimes claimed that there are strong theoretical and empirical reasons for believing that the value of a statistical life declines with age. According to the analysis of this paper, the VSL might increase, be constant, or decline with age or even show a more complicated age-pattern. For example, the VSL might increase with age to peak at a certain age, and then start to decline. Shepard and Zeckhauser (1984) and Rosen (1984) have also noted the possibility of such more complex patterns across the life cycle. Therefore, the claim that there are strong *theoretical* grounds for the view that the VSL declines with age, put forward by, for example, the DG Environment of the EU (2001, p.2), seem premature.

Finally, it remains to investigate how to define and estimate the value of a statistical life in more complex models. For example, intentional bequest motives have been ignored. Chang (1991) introduces such motives in a model with a perfect annuity market, as in Section 3 above. It turns out that the welfare effect of longevity on bequests is ambiguous. Therefore, in the presence of bequest motives of the kind assumed by Chang (1991), the conclusion seems to emerge that the WTP for longevity is also ambiguous. It seems to be an important task for future research to examine the consequences of intentional bequests on the definition of the value of a statistical life. Similarly, it seems important to broaden the analysis so as to cover more general instantaneous utility functions, where, for example, the quality of life is valued. The concept of a Quality Adjusted Life Year, QALY, plays an important role in health economics and is devoted increasing attention also in environmental economics; full or perfect health during a year is attributed the weight 1 while dead is attributed the weight zero. Promising attempts to define the WTP for a QALY are made in Hammitt (2002). Strand (2006) considers the VSL in a model with two health states while alive; healthy and ill, respectively. Such attempts also raise the question how to discount health. should health be discounted in the same way as “money” or in some other constant or non-constant way; see Cairns (2006) for a recent review.

The empirical evidence with respect to the magnitude of the value of a statistical life is mixed, to say the least. Some studies arrive at very modest values while other studies report surprisingly high values. At least partially such differences might reflect real differences, for example, with respect to risk levels, risk changes, income levels, and kind of risk. Moreover, in the case of international comparisons, there might also be cultural and institutional differences between countries. Obviously, income levels as well as risk attitudes also differ between countries.

Appendix

The dynamic budget constraint for the model considered in Section 2 is as follows:

$$\dot{k}(t) = rk(t) + w(t) - c(t) - CV(t) \tag{A.1a}$$

$$k(\tau) = k_\tau \tag{A.1b}$$

$$\lim_{t \to \infty} k(t) e^{-r(t-\tau)} = 0 \tag{A.1c}$$

where $k(t)$ is assets at age t, r is the for simplicity constant rate of interest, $w(t)$ denotes labour (and/or pension) income at age t, $CV(t)$ is a payment made at age t, initially $CV(t)=0$, $\forall t$, and k_τ denotes initial assets. The condition (A.1c), usually referred to as a No Ponzi Game condition, is imposed to prevent unlimited borrowing.

The present value Hamiltonian corresponding to this maximisation problem is:

$$H(t) = u[c(t)] e^{-\theta(t-\tau)} \mu(t;\tau) + \lambda(t;\tau)[rk(t) + w(t) - c(t) - CV(t)] \tag{A.2}$$

where $t \geq \tau$, and $\lambda(t;\tau)$ denotes a costate variable.

In Section 3, the individual is assumed to maximise the remaining expected present value utility, as expressed in equation (8), subject to:

$$\dot{k}(t) = [r + \delta(t) - D\kappa] k(t) + w(t) - c(t) - CV(t) \tag{A.3a}$$

$$k(\tau) = k_\tau \tag{A.3b}$$

$$\lim_{t \to \infty} k(t) e^{-\int_\tau^t (r + \delta(s)(s-\tau))ds + \kappa\varepsilon} = 0 \tag{A.3c}$$

where $D=1$ for $t \in [\tau, \tau+\varepsilon)$ and zero for $t \geq \tau+\varepsilon$. The wealth accumulation equation (A.3a) reflect the fact that the individual receives both interest income and an income from the insurance company. In addition, he receives labour (or possibly pension) income $w(t)$. As before, it is assumed that $CV(t)=0, \ \forall t$. Integrating equation (A.3a), and using the initial condition in equation (A.3b) and the No Ponzi Game condition in equation (A.3c) yields the remaining life time budget constraint:

$$k(\tau) = \int_{\tau}^{\tau+\varepsilon} [c(t) - w(t) + CV(t)] e^{-r(t-\tau)} \mu(t;\tau) e^{\kappa(t-\tau)} dt + \int_{\tau+\varepsilon}^{\infty} [c(t) - w(t) + CV(t)] e^{-r(t-\tau-\varepsilon)} \mu(t;\tau) e^{\kappa\varepsilon} dt \qquad \text{(A.4)}$$

The present value Hamiltonian at age t corresponding to the maximisation problem in equation (8), subject to (A.3), is defined as follows:

$$H(t) = u[c(t)] e^{-\theta(t-\tau)} \mu^{\kappa}(t;\tau) + \eta(t;\tau)[(r + \delta(t) - D\kappa)k(t) + w(t) - c(t) - CV(t)] \qquad \text{(A.5)}$$

where $\eta(t;\tau)$ is a costate variable.

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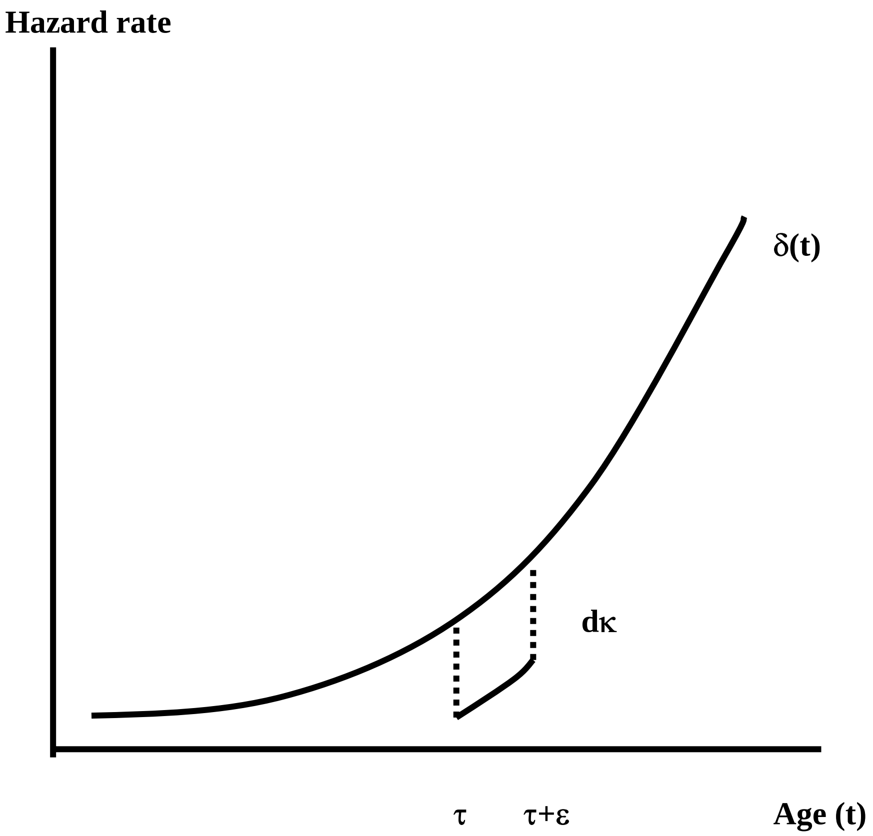

Figure 1. A drop $d\kappa$ in the hazard rate occurring at age τ and lasting until age $\tau+\varepsilon$.

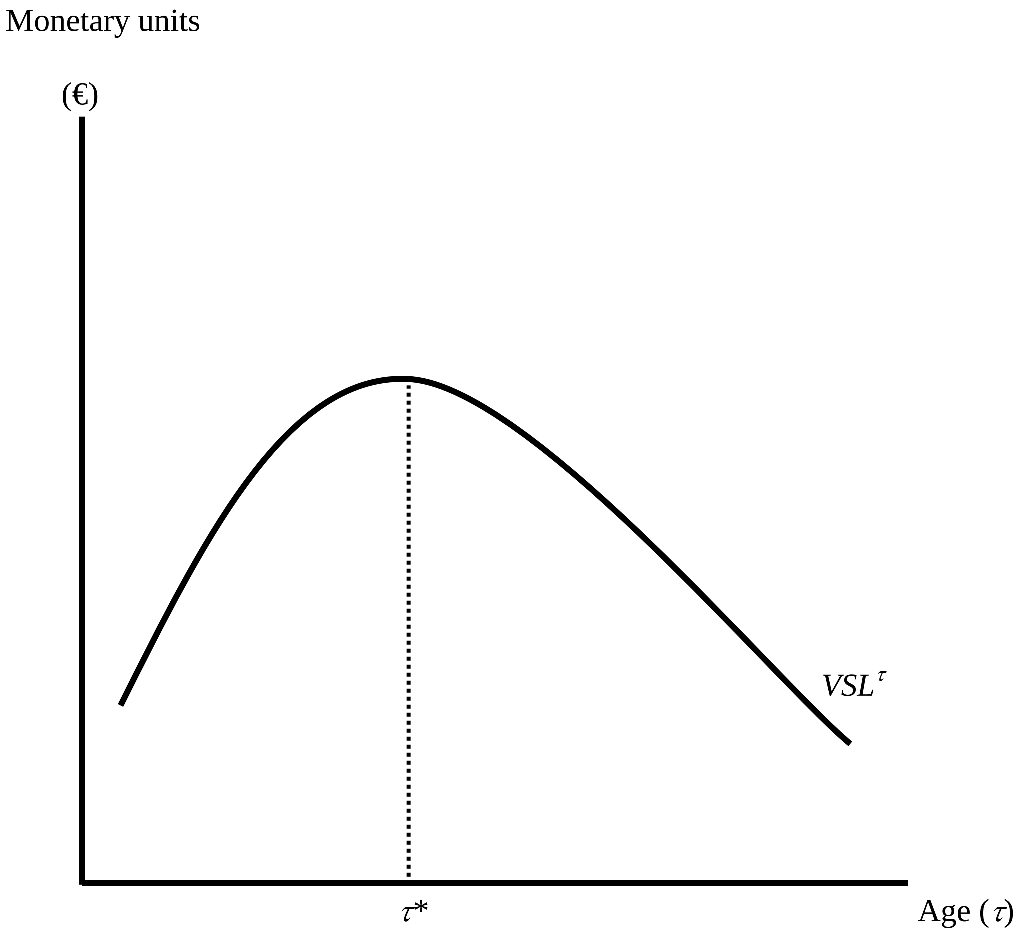

Figure 2. Illustration of a possible age-pattern of the value of a statistical life.

Endnotes

[1]Schelling (1968) and Mishan (1971) advocated willingness-to-pay based approach but did not define the value of a statistical life. Drèze (1962) and Jones-Lee (1969) focus on the willingness to pay for the elimination of a specific risk but the value of preventing a fatality is not defined. I am grateful to Michael Jones-Lee for providing this information in personal correspondence.

[2] Obviously, however, in an intertemporal world with people of different ages, the question is whether the VSL refers to a population mean or an age-specific mean. The definitions used in Sections 2-3 below refer to the value of life of an ex ante homogenous group rather than to (possibly age-specific) population means.

[3] A tontine is a kind of insurance scheme, where the prize is awarded entirely to the participant who survives all the others. It is named after Lorenzo de Tonti, a Neapolitan banker who introduced such a scheme in the middle of the 17th century.

[4] Léonard and Long (1992, pp. 152-154) show that $\lambda^*(\tau;\tau)$ is the worth, or imputed value, of one unit of initial stock of assets, i.e. k_τ; see equation (A.1b) in the Appendix.

[5] Shepard and Zeckhauser (1984) introduce a borrowing constraint. According to Leung (1994, p. 1236) they mistreat this constraint when formulating the individual's decision problem. Leung (1994) shows that a borrowing constraint means that savings must be exhausted at some time before the maximum lifetime. Thereafter, consumption $c(t)$ is equal to $w(t) \geq 0$ at each point in time.

[6] The total effect on the value function of a small change in a parameter is obtained by taking the partial derivative of the present value Hamiltonian (or more generally the Lagrangean), see equation (A.2) in the Appendix, with respect to the parameter, and integrating the result along the optimal path over the planning horizon. For details on this dynamic envelope

theorem, the reader is referred to, for example, Caputo (1990) and LaFrance and Barney (1991).

[7] To illustrate the approach in Figure 1, define the survivor function, conditional on having survived until age τ, as follows: $\mu^{\kappa}(t;\tau) = \mu(t;\tau)e^{\kappa(t-\tau)}$ for $t \in [\tau, \tau+\varepsilon]$ and $\mu^{\kappa}(t;\tau) = \mu(t;\tau)e^{\kappa\varepsilon}$ for $t > \tau+\varepsilon$, where the parameter κ is used to model a change in the hazard rate lasting over the interval $[\tau, \tau+\varepsilon]$. The hazard rate is equal to $\delta(t)-\kappa$ for $t \in [\tau, \tau+\varepsilon)$ and to $\delta(t)$ for $t \geq \tau+\varepsilon$; see Johannesson et al. (1997) for details.

[8] Another recent meta-analysis is undertaken by Miller (2000). His analysis is based on 68 estimates of the value of statistical life. The estimates are collected from 13 countries.

[9] A database was constructed of 203 estimates obtained from 33 studies (since some of the studies reviewed could not be incorporated due to missing information). Multiple observations were drawn from each study if authors reported variations in model specifications or samples from which value of statistical life estimates could be obtained. The mean estimate is around $6 million, with around 50 percent of the observations scattered in the interval running from $1.5 to $8 million.

[10] However, as noted in Section 3, the estimates will be biased unless the risk reduction lasts over a very short time interval.

[11] In fact, most of the studies in Mrozek and Taylor (2001) are also summarised in Viscusi (1992, 1993).

[12] Blomquist (2001b) includes a survey as well as a critical assessment of estimates obtained from studies of self-protection and averting behaviour. Such protective behaviour is evident in motorist choice of automobile type and use of safety equipment such as seat belts.

[13] In the Shepard-Zeckhauser model with actuarially fair annuities the value of a statistical life declines with age. This fact is due to the special assumptions used; compare Section 4 above and Rosen (1988, pp. 296-297).

[14] For a recent and extensive guide to how to undertake CBA of investment projects the reader is referred to Florio (2002).